Linear simulations of acoustic wave propagation in a sun-like spherical shell

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Abstract. Helioseismology is the study of the variations in the internal structure and properties of the dynamics of the sun from measurements of its surface oscillations. We are interested in validating and determining the efficacy of the helioseismic measurement procedure. To this end, we simulate acoustic wave propagation in a solar-like spherical shell that extends from 0.2R to about 1.0004 R, where R is the radius of the sun. In order to render the calculation tractable, wave propagation is treated as a linear phenomenon. In this article, I will discuss the difficulties that are consequent to the assumption of linearity and the methods to resolve them thereof.

Index Terms. Computational acoustics, helioseismology, spherical geometry, wave propagation.

1. Introduction

Solar oscillations possess abundant diagnostic information about the solar interior. Sophisticated observations of these oscillations along with techniques of helioseismology have led to the precise inferences of the solar structure, the rotation-rate and large-scale dynamics. It is important to understand and place bounds on the ability of helioseismology to probe the solar interior. In relation to this, little has been done in the context of the forward problem in spherical domains to complement the extensive inversion analyses applied to data obtained from the Michelson Doppler Imager (MDI) onboard the Solar and Heliospheric Observatory (SOHO), in operation since 1996.

Helioseismic analyses primarily use the line of sight Doppler velocity of plasma at the solar photosphere. This surface is in continual motion due to the interaction, impact and reflection of millions of wave modes. The primary source of wave generation is the intense turbulence present in the convecting uppermost surface layers. In the sun, detected waves that possess diagnostic value are either surface gravity or acoustic modes. While surface gravity modes are constrained to sample only the surface layers, acoustic modes plumb the depths of the solar interior and re-emerge altered by the structure and dynamics of the solar interior. A substantial part of the wave modes that comprise the acoustic wave spectrum travel distances large enough that incorporating sphericity into the model becomes unavoidable.

An accurate description of the solar near-surface layers is highly non-trivial, requiring multi-scale and multi-phenomenon physics. In order to compute the acoustic wave-field in a finite amount of time and keeping in mind constraints of computational expense, we invoke several simplifying assumptions, one of which is the assumption of linearity. Linearity however, comes at the price of being unable to fully model the convection that persists in the outer 30% of the sun. Sustained convection in a medium is possible only if the medium is thermodynamically unstable to convection. Consequently, unchecked by nonlinear terms, the linearized governing equations possess exponentially growing instabilities, created by the thermodynamically unstable solar background model.

In section 2, I will briefly discuss the physics behind convective instabilities and the important properties of the solar near-surface layers, followed by which, in section 3, I shall describe a means to construct a thermodynamically stable near-surface layer that preserves the desired properties. In section 4, results from a numerical simulation are presented and in section 5, the content of this article is summarized.

2. Convective instabilities and the near-surface region

The Brunt-Väisälä frequency, N, is a derived thermodynamic property of a medium that describes the stability of its response to convective motions. It is given by

$$N^2 = g\left(\frac{1}{\Gamma}\frac{\partial \ln p}{\partial r} - \frac{\partial \ln \rho}{\partial r}\right), \tag{1}$$

where ρ is the density of the medium, p is the pressure, g the gravity, and Γ is defined as

$$\Gamma = \left(\frac{\partial \ln p}{\partial \ln \rho}\right)_{ad}, \tag{2}$$

where the ad subscript indicates that the derivative must be calculated along an adiabatic process line. If $N^2 < 0$, the medium is deemed unstably stratified to convection, and motions that cause changes in temperature, density or pressure, act as precursors to convection. The outer 30% of the sun is in a constant state of convection precisely because $N^2 < 0$ in that region. More details regarding the physics of convection and its connection to the Brunt-Väisälä frequency may be found in Christensen-Dalsgaard (2003). In the sun, with the exception of the near-surface layers, convective instabilities are marginal and have timescales of the order of several days. However, the near-surface region represents a more complex problem since the Brunt-Väisälä frequency attains large negative values. Consequently, the instabilities grow quite rapidly, with timescales of the order of a few hours. These instabilities grow exponentially and due to the resultant large magnitudes, tend to reduce the accuracy of the calculation.

It is important to prevent a numerical blow-up arising from the near-surface layers while also keeping in mind that including non-linear terms (that eventually quell any growing instabilities) will result in a substantial increase the computational expense. The idea presented in this paper is that of computing a near-surface region that is stable to convective motions while yet largely preserving other properties of consequence.

The upper-most layers act as broadband reflectors of acoustic waves. This occurs because of the rapidly decreasing scale heights; and therefore the high acoustic cutoff frequencies. In simulations of the solar acoustic wave-field, it is important to preserve the reflective properties of the near-surface layers. The model has to be hydrostatically stable, and must be a smooth extension of the interior state. In order to prevent aphysical reflections, we require that properties such as density, pressure and temperature be continuous and have continuous derivatives. We also require that gravity and the first adiabatic exponent, Γ, to be continuous.

An artificially constructed layer must satisfy all the properties listed above.

3. Constructing an artificial near-surface region

Using a combination of an isothermal near and super-surface layer between $0.9999 \le r \le 1.0007$, placed above a polytrope-like sub-surface structure located between $0.98 \le r < 0.9999$, all the required conditions, stated in section 2, have been satisfied. The polytrope-like sub-surface layer for the region $0.98 \le r \le 0.9998$ is given by,

$$p = p_0\left(\frac{a - r}{0.998989 - r}\right)^{m+1},\ \rho = \rho_0\left(\frac{a - r}{0.998989 - r}\right)^{m}$$

$$m = 2.0358,\ T = T_{(\mathrm{model\ S})},\ \gamma = \max\left(\gamma_0, \frac{m+1}{m}\right)$$

$$a = 0.998989 + 19.1257\,(r - 0.98)^{2.1},\ g = -\frac{1}{\rho}\frac{\partial p}{\partial r} \qquad (3)$$

where r is the radius normalized by the solar radius, γ_0 the ratio of specific heats of model S and γ, the ratio of specific heats of the artificial model. The temperature profile of most of the sub-surface artificial region is chosen to be same as the profile given by the original solar model, which in this case is the model S (Dalsgaard et al., 1996). The ratio of specific heats, γ, is chosen so as to enforce convective stability. On choosing the pressure and density profiles, the gravity is calculated according to the hydrostatic balance equation. For the layer corresponding to $0.9998 < r \le 1.0007$, we place an isothermal atmosphere, given by equations (5.38) and (5.39) of Christensen-Dalsgaard (2003). The pressure scale height term, H, in equations (5.38) and (5.39) is given by H = 90.5 km, and the temperature in this region is $T(0.9998 < r \le 1.0007) = 8727\,K$. We display the properties of the artificial model in Fig. 1 and Fig. 2.

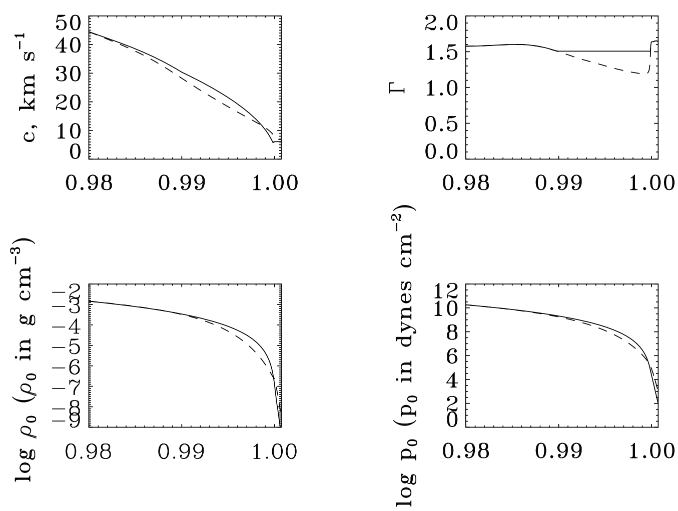

Fig. 1. Plot shows the variation of various thermodynamic properties of the original background model of the solar surface (given by model S of the sun) and the artificial model of the surface. The dashed lines indicate model S and the continuous line indicates the artificial model. Note that density, pressure and sound speed are almost identical in the artificial model and model S.

4. Simulations with the artificial model

The acoustic wave-field is simulated by solving the linearized Euler equations in a spherical shell extending from 0.2R to about 1.0004 R. The numerical techniques and validation procedures employed in this calculation may be found in Hanasoge et al. (2006). The waves in the simulation are constantly excited by radially directed dipoles located very near the surface. In Fig. 3, we demonstrate the stability of the model using the Root Mean Squared (RMS) histories of the latitudinal velocity component. The RMS velocity histories of a simulation with a model S interior coupled with the artificial model of the surface and a simulation purely with model S are compared.

5. Summary and conclusions

A simulation of the solar acoustic wave-field is extremely useful in terms of attempting the forward problem; one can validate a wide range of results from helioseismology. In the past, there have been tests of helioseismology that have involved computations of acoustic wave-fields but none have been performed in spherical geometry; this is indeed the first computation of its kind. In modeling the acoustics, it is important to realize that while the sun has a wide range of scales, from as small as a meter to several million meters, it is not yet computationally feasible to model this entire spectrum. Keeping in mind the important issue of computational feasibility, we have to do away with the physics which that we believe is secondary. As stated earlier, a large saving on computational expense may be achieved when the assumption of linearity is invoked.

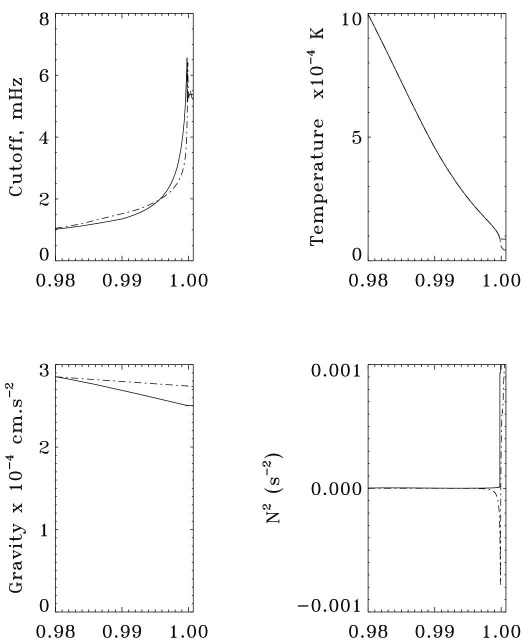

Fig. 2. Dot-dash lines indicate properties derived from model S and continuous lines correspond to the artificial model. The cutoff frequencies of the artificial model match those obtained from model S quite well. For the most part, the temperature profiles are identical. Gravity profiles look somewhat different, although this is not an issue. Most important of all, departing from the behaviour of model S, $N^2 > 0$ over the entire region in the artificial model.

In this paper, it was shown how convective instabilities are an integral part of the solar near-surface and that linearizing the governing equations while still using an accurate solar model creates an unstoppable exponential growth. To avoid the unpleasant consequences of this state of affairs, one may recast the solar near-surface layers to render it convectively stable, while yet preserving its crucial reflective properties. The artificial near-surface model coupled with the model S of the interior has been shown to be stable for long enough that clear helioseismic measurements may be performed. Using a pure model S representation of the sun is shown to be unfeasible when computing the wave-field in the linear limit of the Euler equations.

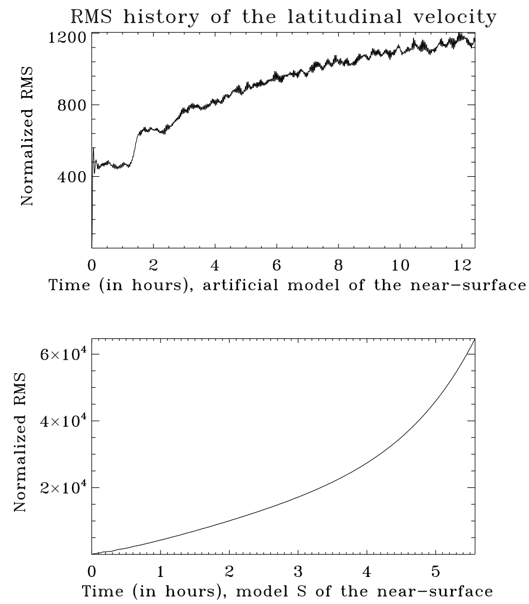

Fig. 3. Comparison between the latitudinal velocity RMS history obtained when simulating with the artificial model of the solar surface and a computation with the model S of the sun. In the lower panel, solving the linearized Euler equations, with model S serving as the background, leads to an exponential growth in the velocity magnitude. The upper panel shows the RMS history of a simulation with the background given by model S in the interior and an artificial model of the surface. It can be seen that the simulation is stable and it was determined that the RMS growth rate of the velocity is sub-linear.

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