Dynamo Processes

Dibyendu Nandy

Abstract Magnetic fields play an important role in defining and modulating the space environment within the heliosphere. How these magnetic fields originate and evolve in stars such as the Sun or planets such as the Earth, is in itself a compelling question – the answer to which is of universal importance to astrophysics and space science. It is thought that magnetic fields are created through a dynamo process which involves complex, non-linear interactions within the magnetized plasma or fluid that is often encountered in astrophysical systems. In this chapter, after briefly discussing the importance of magnetic fields in the heliosphere, we provide a gentle introduction to concepts in magnetohydrodynamics, describe the mathematical foundation of mean-field dynamo theory, and explain the basic physical processes that constitute the dynamo mechanism. In doing this, we focus mainly on our star – the Sun – which is the primary source of heliospheric magnetic fields and their variability.

1 Introduction: Magnetic Fields in the Heliosphere

The heliosphere – or the sphere of influence of the Sun – encompasses the solar system including all its planets. The environment within this heliosphere is primarily governed by the radiative, particulate, and magnetic output of the Sun. Slow long-term variation in the Sun's radiative and magnetic output influences planetary atmospheres and climate. For example, a period of reduced solar activity between 1645-1715 AD, known as the Maunder minimum, coincided with a period of global cooling known as the *Little Ice Age*. On shorter timescales, explosive solar events such as flares and Coronal Mass Ejections (CMEs) hurl vast amounts of magnetized plasma into space; these adversely affect satellite operations, telecommunication fa-

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cilities, oil-pipelines and air-traffic on polar routes on the one hand, but generates beautiful auroras on the other hand. Understanding solar variability and the changes that it induces in the heliosphere are therefore of particular importance for planetary atmospheres, our technologies in space and on Earth and our life and society in general.

Much of the solar phenomena that defines and modulates the heliosphere can be traced back to the presence of magnetic fields in the Sun. These magnetic fields are ever-changing. On relatively short timescales, of seconds to days, sudden magnetic re-organization, mediated through magnetic reconnection leads to eruptive processes such as flares and CMEs. On moderate timescales, the Sun's magnetic output varies periodically, with an average period of 11 years. On relatively longer timescales of centuries to millennia, the magnetic (and the coupled radiative) output varies in a non-periodic irregular manner. It is widely believed that this solar magnetic variability, on timescales stretching from years to stellar evolutionary timescales (on the order of billions of years), has its origin in a dynamo mechanism in the Sun's interior. Other solar-like stars are also known to have variable magnetic activity – most likely due to the action of a similar dynamo mechanism in their interiors.

Some planets, such as the Earth, have magnetic fields. The Earth's magnetic field is much weaker compared to the Sun but it plays an important role in maintaining the magnetosphere – which shields us from harmful space radiation, plasma and energetic particles such as cosmic rays. Even though the Earth's magnetic field appears relatively non-varying, it is known to have reversed direction (flipping of the magnetic polarities) many times in the past. Moreover, if the Earth's magnetic fields were not being continually generated, it would possibly have decayed on a timescale of $20,000$ years. This points out that a dynamo mechanism is working in the Earth's interior, creating and changing the geomagnetic field. It is quite likely that life as we know it, would not have been possible without the shielding and protective effects of the geomagnetic field which owes its existence to the geo-dynamo.

The earlier discussion illuminates the important role that dynamo processes play in the heliosphere, both in determining solar variability and in maintaining the geomagnetic field. Our focus here is on understanding the physical processes that underlie this dynamo mechanism. The rest of this chapter is organized as follows. In Section 2 we introduce the subject of magnetohydrodynamics and outline key physical concepts that are critical to understanding the dynamo mechanism. In Section 3 we lay the mathematical basis of dynamo theory through the dynamo equations and describe how they are derived. In Section 4 we focus on the solar dynamo as a representative example of dynamo action in the heliosphere. Finally, we end with some concluding remarks in Section 7.

2 Magnetohydrodynamics: Basic Theoretical Ideas

At about the same time that the magnetic nature of the sunspot cycle was being established through observations in the early 20th Century, efforts were on to construct a theory to describe the behavior of magnetic fields in a plasma. This theory, which came to be known as magnetohydrodynamics (MHD), was pioneered by Hannes Alvén – who was eventually awarded the Nobel Prize in 1970 for his contributions to MHD. Some basic concepts in MHD are crucial for gaining an insight into the dynamo mechanism and understanding many observable features of the solar magnetic cycle; it is therefore logical to start with these concepts. Assuming that readers have some background knowledge of Electromagnetism, let us begin with the Maxwell's equations which describe electromagnetic fields and relate them to their sources. Two equations are of particular importance here:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \tag{2}$$

Here E and B denotes electric and magnetic fields, respectively, t denotes time, μ_0 the permeability of free space and J the current density. In Equation 2 above we have neglected the displacement current term (assuming that we are dealing with a non-relativistic system). A further equation of relevance here is the one that expresses Ohm's law in a conductor (of conductivity σ) moving with a velocity v relative to the magnetic field:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{3}$$

Substituting for E from Equation 3 in Equation 1, we have

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \frac{\mathbf{J}}{\sigma}). \tag{4}$$

Now, substituting for J from Equation 2 in Equation 4 and using the relationship $\lambda = 1/\mu_0\sigma$ (where λ is the magnetic diffusivity), we get:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times B - \lambda \nabla \times \mathbf{B}). \tag{5}$$

If magnetic diffusivity is constant in space, Equation 5 simplifies to:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \lambda \nabla^2 \mathbf{B}. \tag{6}$$

What we have ended up deriving here is known as the induction equation (expressed in different forms in Equations 5 and 6). This is one of the most fundamental equations in MHD and it describes the behavior of magnetic fields in a plasma system.

In deriving the induction equation, some assumptions are made related to the system in which this equation is applicable. These assumptions are:

- The plasma is a continuum – valid when the typical length scale of the system exceeds the ion gyro-radius
- The plasma is a single fluid – valid if the system length scale is much larger than the Debye shielding length
- Plasma is in thermodynamic equilibrium with distribution close to Maxwellian – valid if system timescale exceeds typical collision timescale and length scale exceeds mean free path
- Relativistic effects are unimportant – holds if typical systems (flow) speeds are much less than the speed of light
- Permeability, and in certain cases conductivity or diffusivity are isotropic

Another important equation for the MHD system describes how the velocity field $\mathbf{v}$ evolves (the equation of motion):

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla\left(p + \frac{B^2}{8\pi}\right) + \frac{(\mathbf{B}.\nabla)\mathbf{B}}{4\pi\rho} + \mathbf{g} + \nu\nabla^2\mathbf{v}, \tag{7}$$

where ρ is the density, p the pressure, g the gravitational field and ν the kinematic viscosity of the plasma. Note that Equation 7 is the familiar Navier-Stokes equation of hydrodynamics with two additional terms in it due to the inclusion of magnetic forces. The first additional term $B^2/8\pi$ is the pressure contribution from the magnetic field and the second term $(\mathbf{B}.\nabla)\mathbf{B}/4\pi\rho$ signifies tension along the magnetic field lines.

The induction equation (Equation 5 or 6), the equation of motion (Equation 7), the energy equation for the system

$$\frac{\partial p}{\partial t} + (\mathbf{v}\cdot\nabla)p + \gamma p\nabla\cdot\mathbf{v} = Q, \tag{8}$$

where γ is the adiabatic coefficient and the term Q encompasses effects of heating, cooling and conduction (including ohmic heating), the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0, \tag{9}$$

and

$$\nabla\cdot\mathbf{B} = 0, \tag{10}$$

provides a complete description of the MHD system.

2.1 Magnetic Reynolds Number

The ratio of the first term to the second term on the R.H.S. of the induction equation (Equation 6) is

$$R_m = \frac{VB/L}{\lambda B/L^2} = \frac{VL}{\lambda} = \frac{\tau_\lambda}{\tau_V}. \tag{11}$$

The dimensionless number, R_m, is known as the magnetic Reynolds number and V, B, L are the typical values of velocity, magnetic field and length scale of the system under consideration (τ_λ and τ_V are typical diffusion and velocity timescales). One aspect which is immediately obvious is that R_m, which is proportional to L, will be larger by orders of magnitude in astrophysical systems relative to laboratory systems. Therefore magnetic fields in astrophysical systems behave very differently from their laboratory counterparts. For example, it is very difficult to build a laboratory model of a dynamo that is sustained through plasma motions acting to counter dissipation (through diffusion). However, significant progress have been made in the last decade in building laboratory dynamos and in the month of November, 1999, magnetic field self-excitation and a growing dynamo mode was observed for the very first time in any laboratory at the Riga Dynamo Facility in Latvia (Gailitis et al. 2000).

2.2 Magnetic Diffusion

If we consider the extreme limit, $R_m \ll 1$, then we can ignore the first term on the R.H.S. of Equation 6. In that case we are left with:

$$\frac{\partial \mathbf{B}}{\partial t} = \lambda \nabla^2 \mathbf{B}. \tag{12}$$

This is the magnetic diffusion equation and expresses the fact that in the absence of any source or forcing term (involving plasma motions and associated energy), the initial magnetic field will simply diffuse away (with any inhomogeneity being smoothed out) with a characteristic timescale of $\tau_d = L^2/\lambda$ over a typical length scale of L.

2.3 Concept of Flux Freezing

If $R_m \gg 1$, as in most astrophysical systems, then the diffusion term in Equation 6 is relatively unimportant compared to the preceding term. When one considers the

ideal MHD limit (infinite conductivity and therefore very high R_m) and drops the diffusion term altogether, Equation 6 reduces to:

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B), \tag{13}$$

in which case it can be shown that (see e.g., Choudhuri 1998)

$$\frac{d}{dt}\int_S B \cdot dS = 0. \tag{14}$$

The above equation describes the following physical phenomenon: If the magnetic field vector in a plasma system satisfies Equation 13, then the magnetic flux through any surface (say, S) constituting a part of the moving fluid will remain time-invariant when that fluid element is moving. In other words, magnetic fields remain frozen to the flow and moves along with it.

This is known as Alfvén's theorem of flux-freezing (Alfvén 1942) and can be appreciated by recognizing that if $R_m \gg 1$, as in most astrophysical systems, then magnetic fields attached to a moving fluid parcel do not have enough time to be dissipated or dispersed through diffusion (think $\tau_\lambda/\tau_V \gg 1$, where $\tau_v = L/v$ is the flow timescale). Therefore the frozen-in magnetic field moves with the fluid and in situations where the plasma-β parameter (ratio of gas to magnetic pressure) is high, allows the fluid to distort it. This is a fundamental concept in MHD and implies that any twisting or stretching motion in a magnetized plasma will result in the magnetic field being twisted or stretched.

2.4 Magnetic Buoyancy

Now let us look at the stability of horizontal magnetic flux tube in a system with gravity as in an astrophysical object. Suppose that the gas pressure outside the flux tube is $p_{external}$, the gas pressure inside is $p_{internal}$ and the flux tube has a magnetic field strength B and thus a magnetic pressure $B^2/8\pi$ associated with it. Then for pressure balance across the bounding surface of the flux tube:

$$p_{external} = p_{internal} + \frac{B^2}{8\pi}. \tag{15}$$

It follows from the above equation that

$$p_{internal} < p_{external}. \tag{16}$$

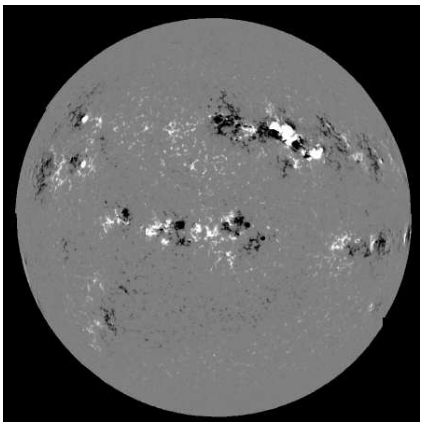

Fig. 1 A magnetogram image of the solar surface showing bipolar sunspot pairs. White denotes positive polarity while black denoted negative polarity sunspots. Note how the bipolar sunspot pairs have opposite polarity orientation in the two hemispheres with positive polarity sunspots appearing on the right hand side (i.e., on the leading side in the direction of rotation – left to right) above the equator and on the left (following side) below the equator. The small black and white patches and the gray region outside of sunspots, denote the much weaker, diffuse field on the surface. The formation of sunspots can be explained based on the concept of magnetic buoyancy.

Since pressure $p = \rho RT$ (where R is the universal gas constant and T is the temperature) the above relation implies that for a magnetic flux tube

$$\rho_{internal} RT_{internal} < \rho_{external} RT_{external} \tag{17}$$

In situations where the plasma in the interior and exterior of the flux tube are in isothermal condition, this means the density inside the flux tube would be less than the density outside.

Therefore, any strong magnetic flux tube in the interior of astrophysical systems, would likely be less dense that its ambient surrounding. In the presence of a gravitation field directed downward, this would translate to the flux tube being buoyant (follows simply from Archimedes' principle). The flux tube would then tend to rise up against gravity. This concept of magnetic buoyancy, which was first worked out by Eugene N. Parker (1955a), explains the formation of bipolar sunspot pairs that are observed in the Sun (Figure 1).

3 Formulating of the Kinematic Mean-Field Dynamo Problem

Plasma systems in astrophysical objects such as stars and planets often exist in a state of vigorous turbulence and host large scale flows. This is especially true for the those systems where convection, driven by temperature gradients, can support plasma motions. The kinetic energy of the plasma in such a region can presumably be used to feed a mechanism that converts it to magnetic energy. The subject that attempts to understand this mechanism and seeks theoretical explanations for the origin and evolution of astrophysical magnetic fields (or for that matter laboratory magnetic fields - as long as it is generated in a plasma), is known as dynamo theory.

To address the dynamo generation of magnetic fields one would have ideally liked to solve the complete set of MHD equations (Equations 5-10) simultaneously, to search for an answer to how interacting velocity and magnetic fields sustain each other and generate a global magnetic field. That, however, is an extremely formidable task that is highly computer intensive and relatively less suitable for a physical appreciation of individual processes at work in the system. We restrict ourselves to the more tractable problem of kinematic dynamo theory – where the velocity field is prescribed and one solves Equation 5 or 6 for the magnetic field evolution.

The first significant effort in this subject turned out to be a negative result known as the *anti-dynamo theorem*. T. G. Cowling (1934) showed that an axisymmetric velocity field (the simplest possible flow field in a astrophysical body) cannot sustain an axisymmetric magnetic field (that is often observed). Arguably the most important step in astrophysical dynamo theory was taken by Parker (1955b) when he proposed that helical turbulent motions within rotating plasma systems (which are inherently non-axisymmetric) can sustain a magnetic field.

Parker's original dynamo formulation was focussed towards explaining the origin of the solar cycle and was largely intuitive. About a decade later, Steenbeck, Krause and Rädler (1966) put Parker's ideas on a firm mathematical footing – developing what is today known as the mean field theory of magnetohydrodynamics. Essentially they showed that a crucial term for dynamo action - the mean-electromotive-force (*mean e.m.f.* – essentially that process which converts the energy of plasma motions to magnetic energy) – is generated as a result of averaging over the interactions of the turbulent (or fluctuating) parts of the velocity and magnetic fields in a plasma.

In a turbulent system, the velocity and magnetic field can be written in terms of the average and fluctuating parts:

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}', \quad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'. \tag{18}$$

Where the over-line indicates average and prime indicates fluctuating parts. Substituting Equation 18 in the general form of the induction equation (Equation 5) and averaging the terms we get

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}}) + \nabla \times \mathscr{E} - \nabla \times (\lambda \nabla \times \overline{\mathbf{B}}), \tag{19}$$

considering that $\overline{\mathbf{v}'} = \overline{\mathbf{B}'} = 0$. Here $\mathscr{E}$ is the *mean e.m.f* term given by:

$$\mathscr{E} = \overline{\mathbf{v}' \times \mathbf{B}'}. \tag{20}$$

The *mean e.m.f* term can be perturbatively evaluated using a scheme known as the *first order smoothing approximation* (which requires a severe and possibly questionable truncation of the series expansion associated with the *mean e.m.f* term.). Assuming that the turbulence is isotropic, this leads to

$$\mathscr{E} = \alpha \overline{\mathbf{B}} - \beta \nabla \times \overline{\mathbf{B}}, \tag{21}$$

where

$$\alpha = -\frac{\tau}{3} \overline{\mathbf{v}'.(\nabla \times \mathbf{v}')} \tag{22}$$

and

$$\beta = \frac{\tau}{3} \overline{\mathbf{v}'.\mathbf{v}'}. \tag{23}$$

The term α, which is at the heart of the dynamo α-effect – represents the average helical motions present in the plasma, τ is the correlation time for turbulence and β signifies turbulent diffusivity. On substituting Equation 21 for the *mean e.m.f.* in Equation 19 and considering that the net magnetic diffusivity, η, is given by

$$\eta = \lambda + \beta, \tag{24}$$

we finally get

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}}) + \nabla \times (\alpha \overline{\mathbf{B}}) - \nabla \times (\eta \nabla \times \overline{\mathbf{B}}). \tag{25}$$

Usually the turbulent diffusivity β is much greater than λ and in all subsequent analysis from now on, we are going to denote the net magnetic diffusivity by the term η. Equation 25 is known as the dynamo equation and describes the dynamo generation of magnetic fields.

3.1 Evolution Equations for the Poloidal and Toroidal Fields

Most astrophysical objects of interest in dynamo theory, e.g., stars or planets are spherical systems, which by virtue of their rotation, also sustain mostly axisymmetric large-scale velocity and magnetic fields (although helical turbulent motions are not axisymmetric, their net effect, averaged over large-scales can result in the creation of an axisymmetric field). It is advantageous then to formulate the dynamo problem in terms of axisymmetric magnetic and velocity fields in spherical polar

coordinates, expressed as

$$\mathbf{B} = B(r,\theta,t)\,\mathbf{e}_\phi + \nabla \times \left[A(r,\theta,t)\,\mathbf{e}_\phi\right], \tag{26}$$

$$\mathbf{v} = \mathbf{v}_p\,(r,\theta) + r\sin\theta\,\Omega\,(r,\theta)\,\mathbf{e}_\phi\,. \tag{27}$$

where B represents the toroidal component of the magnetic field (i.e., in the direction of rotation), $\mathbf{B}_p = \nabla \times (A\mathbf{e}_\phi)$ the poloidal component (in the $r-\theta$ meridional plane) and A the vector potential for the poloidal field; Ω is the (rotational) angular velocity (in the ϕ direction), and $\mathbf{v}_p = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta$ is the meridional circulation (in the $r-\theta$ plane). We substitute Equations 26 and 27 in the dynamo equation (Equation 25), to get the two coupled equations:

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\mathbf{v}_p.\nabla)(sA) = \eta\left(\nabla^2 - \frac{1}{s^2}\right)A + \alpha B, \tag{28}$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_rB) + \frac{\partial}{\partial \theta}(v_\theta B)\right] + \nabla\eta \times (\nabla \times B) = \eta\left(\nabla^2 - \frac{1}{s^2}\right)B + s(\mathbf{B}_p.\nabla)\Omega + \nabla \times (\alpha\mathbf{B}_p), \tag{29}$$

where $s = rsin\theta$. Equation 28 and Equations 29 are the evolution equations for the poloidal and toroidal components of the magnetic field, respectively, which have to be solved with the specification of boundary conditions appropriate for the system under consideration. Often an amplitude-limiting quenching factor, that signifies the non-linear feedback of the magnetic fields on the flows, is also included through a parametrization of the α-effect such as $\alpha \sim \alpha_0/[1+(B/B_{quenching})^2]$, where α_0 is the amplitude of the α-effect and $B_{quenching}$ is the field strength at which non-linear feedbacks become important.

Let us now discuss specific terms of the toroidal and poloidal field evolution equations to gain an insight into what physical processes they represent. The first terms on the L.H.S. of Equations 28-29 describe how the magnetic field evolves in time. This time evolution depends on the other (following) terms in the equations.

Advective Flux Transport by Meridional Circulation: The second terms on the L.H.S. of the above equations represent the transport of magnetic flux by meridional circulation, including stretching or compression effects due to expanding or contracting flow fields. In the high magnetic Reynolds number regime, where the magnetic flux in frozen in the plasma, these terms are quite important and magnetic field transport is dominated by plasma flows – characterizing a regime often referred to as the advection-dominated or circulation dominated regime.

Flux Transport Due to Diffusion Gradients: The third term on the L.H.S. of Equation 29 involving the gradient of η represents a form of magnetic transport (also referred to as diamagnetic transport) in which the magnetic field is carried by a velocity field (the latter's magnitude being proportional to the gradient of diffusion)

whose net effect is to carry magnetic fields from regions of strong diffusivity to regions of lower diffusivity. Obviously, in the absence of any diffusivity gradient (i.e., when η is constant in space), this term vanishes.

Decay and Dispersal by Magnetic Diffusion: The first terms on the R.H.S. of the above equations denote the effect of diffusivity on the magnetic field. Note that diffusion disperses the field in such a way as to smooth out inhomogeneity. Therefore diffusion does play a role in transporting magnetic fields in such systems. The relative importance of diffusive flux transport is measured in terms of the magnetic Reynolds number again; when R_m is low (equivalently when diffusive timescales are shorter than the meridional flow timescale), diffusive dispersal dominates over advective flux transport. However, as opposed to flux transport by meridional circulation, diffusion also destroys the field (while being dispersed); this decay happens over a diffusion timescale, τ_d, which we have already discussed before.

Dynamo Source Terms: There are three important source terms in the above equations which represent the creation of magnetic field, without which the magnetic field of the system will simply decay away. These terms are described below.

The Ω-effect Toroidal Field Source: The second term on the R.H.S. of Equation 29 represents the stretching of poloidal field by non-uniform rotation to create toroidal field. This process can be visualized in the following way. Imagine a pre-existing poloidal field (in the $r-\theta$ plane) that is frozen in a plasma system which is rotating at different speeds in the ϕ direction. Then that part of the poloidal field which is rotating faster will be stretched out of the $r-\theta$ plane and into the ϕ direction, therefore generating a new toroidal field component. This toroidal field generation process is commonly referred to as the Ω-effect because it owes its existence to the presence of non-uniform rotation.

Mean-Field α-effect Sources: The last terms on the R.H.S. of the above equations, involving α (derived from the *mean e.m.f.*), denotes the mean-field source terms. The α-effect term in Equation 28 denotes the twisting of toroidal field out of the ϕ direction and into the $r-\theta$ plane creating a new poloidal component. While the α-effect term in Equation 29 denotes the twisting of poloidal field out of the $r-\theta$ plane and into the ϕ direction therefore creating toroidal field. Obviously, the presence of helical motions is essential for these terms to exist.

3.2 Efficiency of the Dynamo Process: Dynamo Numbers

The efficiency of the dynamo mechanism can be estimated by the relative importance of the source and diffusive terms in the above equations. First lets consider the relative importance of the two source terms on the R.H.S. of Equation 29 as compared to the diffusive term through a simple dimensional analysis. Considering that $\nabla \sim 1/L$ (where L is the typical length-scale of the system) the ratio of the Ω-effect source term (second term on the R.H.S. of Equation 29) to the diffusive term (first

term on the R.H.S.) is

$$C_\Omega = \frac{(\Delta\Omega)L^2}{\eta} \tag{30}$$

and the ratio of the α-effect source term (third term on the R.H.S. of Equation 29) to the diffusive term (first term on the R.H.S.) is

$$C_\alpha = \frac{\alpha L}{\eta}. \tag{31}$$

Here, $\Delta\Omega$ represents the difference in angular velocity over the typical length-scale L, and α and η denote typical magnitudes of the α-effect and diffusivity, respectively, for the system.

When $C_\Omega \ll C_\alpha$ (weak differential rotation), the Ω-effect source term (due to differential rotation) in Equation 29 can be neglected and the only remaining source terms in the poloidal and toroidal field evolution equations are both related to the α-effect and the resultant dynamo system is known as a α^2 dynamo. When C_Ω is comparable to C_α, we have to keep all the three source terms and the resultant dynamo system is known as a $\alpha^2\Omega$ dynamo. In the other extreme, when $C_\Omega \gg C_\alpha$ (strong differential rotation), we can neglect the α-effect source term in Equation 29 (but not in Equation 28, where it is still the only source term!) and the resultant system is known as a $\alpha\Omega$ dynamo. The Sun has strong differential rotation, and therefore, the solar dynamo is believed to be of the $\alpha\Omega$ type.

If we take the product of these two dimensionless numbers C_α and C_Ω, we get what is most commonly referred to as the dynamo number:

$$N_D = \frac{\alpha(\Delta\Omega)L^3}{\eta^2} \tag{32}$$

The dynamo number compares the relative efficiency of the source terms to the diffusive terms in the dynamo equations. A high dynamo number ($N_D \gg 1$) for a specific system indicates that the dynamo process is efficient at creating magnetic fields in that system. On the other hand, a low dynamo number ($N_D < 1$) indicates that diffusion dominates over the source terms and therefore dynamo action is unlikely in that system.

4 Application to the Sun: The Solar Dynamo

The Sun's magnetic field and its variability is the primary determinant of the electromagnetic environment of the heliosphere. The behavior of solar magnetic fields are also well documented and the Sun's internal properties well constrained. It is pru-

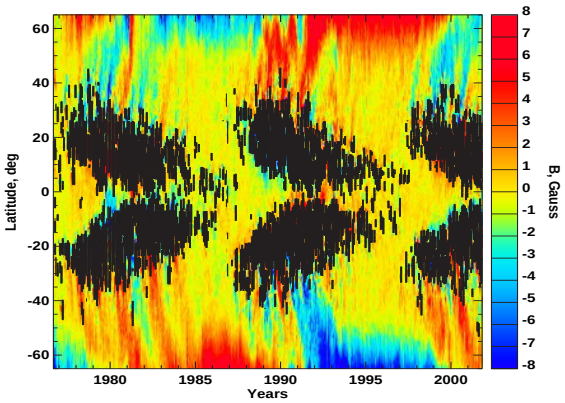

Fig. 2 Latitude versus time plot of sunspots as well as the weak, diffuse field. The time evolution of longitude-averaged weak, diffuse field is shown with colors depicting the magnitude of the field. Overlaid on this are vertical black lines which connect the latitudinal distribution of sunspots at a given time. This is often referred to as the sunspot butterfly diagram, as much an example of the wonderful symmetry sometimes found in nature, as of our fixation with something beautiful. The first plot, depicting the variation of sunspot eruption latitude with time in this manner, was constructed by Maunder (1904). *Image courtesy: Alexander G. Kosovichev.*

dent then to take the Sun's magnetic cycle as an example, and apply our knowledge of MHD dynamo theory to study it.

4.1 Solar Magnetic Fields

Sunspots have been observed for centuries, with telescopic observations initiated by Galileo Galilei in the early 1600s. The magnetic nature of sunspots have also been thoroughly explored since the discovery of strong magnetic fields, on the order of 1000 Gauss (G), within sunspots by G.E. Hale in the early 20th century (Hale 1908). Sunspots often appear in bipolar pairs (known as solar active regions) whose polarity orientation is opposite in the two hemispheres. These bipolar sunspot pairs have a systematic tilt that increases with latitude in both the hemispheres – a phenomenon termed as Joy's law (Hale et al. 1919). Many of these properties can be identified in Figure 1. Years of sunspot observations have now firmly established that the sunspot cycle has an average period of 11 years, with large amplitude fluctuations. At the beginning of a new cycle, sunspots start appearing at about 40° latitude and with the progress of the cycle more and more sunspots appear at lower and lower latitudes. At the end of 11 years, a new cycle starts with bipolar sunspot pairs which have opposite magnetic polarity to that of the previous cycle, appearing again at about 40° latitudes. Thus, if one considers not only the number of sunspots but also their sign, the full cycle constitutes 22 years. Only after the development of the magnetograph by Babcock and Babcock (1955), it became possible to study the much weaker

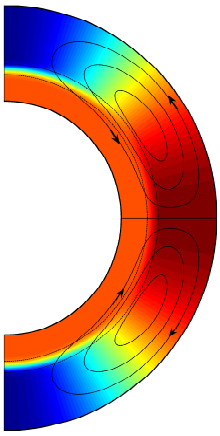

Fig. 3 Some representative streamlines of the solar meridional circulation are shown in black contours with arrows indicating the direction of flow. The background in color depicts an analytic fit to the solar internal rotation profile. Deep red indicates faster rotation, while blue indicates slower rotation. The two concentric dashed lines indicate the boundary of the tachocline – the region of strong radial shear in the rotation near the base of the solar convection zone. The domain shown is a meridional cut of the solar interior extending from $0.55R_\odot$ to $R_\odot$ (where $R_\odot$ is the solar radius) and from the north (top) to the south (bottom) pole.

magnetic field that is distributed on the solar surface outside of sunspots. This field is of the order of 10 G and is concentrated in large unipolar patches (possibly within unresolved magnetic fibrils) that migrate poleward with the progress of the solar cycle (Bumba and Howard 1965; Howard and LaBonte 1981). This weak, diffuse field reverses its polarity (near the poles) at the time of solar maximum (the phase when the maximum number of sunspots are seen on the solar surface). It turns out that this polarity reversal also happens at intervals of 11 years and this shows that the cycle of the weak, diffuse field, is intimately connected to the sunspot cycle.

Figure 2 captures both the sunspot cycle as well as the cycle of the weak, diffuse field in one unified picture. The equatorward propagation of the sunspot formation belt as well as the poleward propagation of the weak, diffuse field is clearly discernable. Note also that the weak, diffuse field near the poles reverses its polarity at the time when the maximum number of sunspots are seen on the solar surface.

4.2 Large-Scale Solar Flows

Observations of small magnetic features on the solar surface show that they seem to be “carried” by a axisymmetric surface flow. An estimate of this surface flow velocity is possible by following the motions of these surface features and doppler shift measurements. These estimates point out that there is a markedly poleward

directed flow (from the equator) on the surface of the Sun. This largely axisymmetric flow in the meridian plane is referred to as the meridional circulation. With the development of helioseismic techniques such as ring-diagram analysis and time-distance helioseismology, it has became possible to study this flow more accurately. It is found that this poleward meridional flow pervades till at least, the outer 15% of the Sun (Giles et al. 1997). Various studies place the magnitude of this flow somewhere in between 10 m s^{-1} – 25 m s^{-1}. While, as yet, it has not been possible to observe a meridional flow in the solar interior, it follows from mass conservation that there should be an equatorward counter-flow somewhere near the base of the SCZ. Figure 3 shows a possible profile of this meridional circulation in the solar interior, although, at this writing, there is no consensus on the exact location of the equatorward counterflow.

It has been known for years that the Sun rotates differentially with the equator rotating faster than the poles. Helioseismology has now mapped the internal solar rotation as well (Charbonneau et al. 1999). The rotation varies with latitude in the upper 30% of the Sun - with the rotation rate increasing with decreasing latitude. Below the base of the solar convection zone (SCZ) this latitudinal variation in the rotation changes over into a radial variation over a thin region - termed as the tachocline. Beneath the tachocline, the radial variation falls off, with the radiative interior rotating more or less uniformly (see Figure 3 for an analytic fit to the helioseismically determined solar internal rotation profile).

4.3 Understanding the Solar Cycle

Having described the observable properties of the solar cycle lets now summarize the current state of our understanding of the solar dynamo mechanism. For illustrative purposes, lets assume that we have a large-scale north-south oriented poloidal magnetic field in the solar interior. Due to the differential rotation, specifically the equator rotation faster than the poles, this poloidal field would be stretched out more near the equator (in the direction of rotation) creating a toroidal field. It is easy to see that this stretching would create opposite directed toroidal fields in the two hemispheres. With the discovery of the tachocline – the region of strong radial shear at the base of the SCZ, it is now fairly certain that the strong toroidal component of the solar magnetic fields are stored and amplified there. Given that the tachocline lies in a region of sub-adiabatic temperature gradient (also known as the overshoot layer), where magnetic buoyancy is suppressed, this storage of magnetic fields is possible there (Spiegel & Weiss 1980; van Ballegooijen 1982).

Very strong magnetic flux tubes, perhaps mediated via overshooting convection, can however come out of this “stable” layer. Once out in the convection zone, these magnetic flux tubes are subject to magnetic buoyancy (a concept we have outlined earlier in Section 2.4) and they rise up radially to erupt through the surface as bipolar sunspot pairs. Obviously in one of the spots the magnetic field is directed outward while in the other it is directed inward and thence their bipolar nature. Also, since the

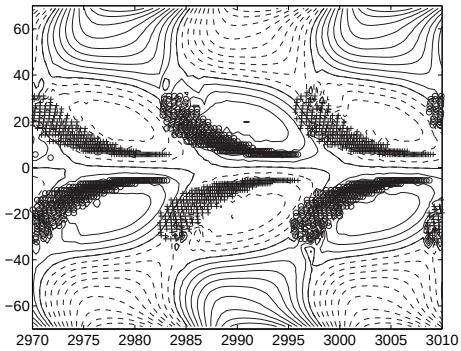

Fig. 4 Theoretically simulated solar butterfly diagram (y-axis denotes solar latitude and x-axis denotes time in years) from a recent dynamo model based on the Babcock-Leighton idea for poloidal field generation (from Chatterjee, Nandy & Choudhuri 2004). The background shows contours of diffuse radial field. The dashed contours are for negative radial field, whereas the solid contours are for positive radial field. Sunspot eruption latitudes are denoted by symbols "o" and "+", indicating negative and positive toroidal field respectively. Note that many features of the observed solar butterfly diagram (Figure 2) are well-reproduced by this model.

underlying toroidal field is directed in opposite directions in the two hemispheres, bipolar sunspot pairs have opposite orientations in the northern and southern hemispheres. Since the Sun is a rotating system, the buoyantly rising flux tubes are subject to the Coriolis force which results in the axis of the flux tubes acquiring a tilt which is manifested in the observed tilt of bipolar sunspot pairs (see Figure 1).

It was initially thought that helical turbulent convection acts on the buoyantly rising toroidal flux tube to twist them into the $r-\theta$ plane re-generating the poloidal component of the magnetic field and this completing the dynamo chain (Parker 1955b). However, simulations of the buoyant rise of toroidal flux tubes show that the field strength of these flux tubes have to be on the order of 10^5 G at the base of the SCZ to match observable properties such as emergence latitudes (Choudhuri & Gilman 1987) and tilt angle distribution (D'Silva & Choudhuri 1993; Fan, Fisher & DeLuca 1993). This field strength is an order of magnitude stronger than the equipartition magnetic field strength (at which the energy in the fields and convection are in equipartition). This suggests that the mean-field dynamo α-effect powered by helical turbulent convection (discussed in Section 3) will get quenched and be ineffective in twisting such strong fields. Therefore alternative possibilities have to be explored. One of these alternative is an idea originally proposed by Babcock (1961) and Leighton (1969); the decay of tilted bipolar sunspot pairs, mediated via diffusion and meridional circulation would preferentially carry a net flux to the polar regions, reversing the older polar field and building up a new cycle poloidal field. This effect is actually observed, and has been extensively used in used in recent dynamo models to explain many observed features of the solar cycle (see e.g., Choudhuri, Schüssler & Dikpati 1995; Durney 1995; Dikpati & Charbonneau 1999;

Nandy & Choudhuri 2002; Chatterjee, Nandy & Choudhuri 2004 and Yeates, Nandy & Mackay 2008).

These dynamo models based on the Babcock-Leighton idea solve the two coupled evolution equations for the poloidal and toroidal fields (Equations 28 and 29) in the $\alpha\Omega$ regime. However, both the mean-field source terms (the last terms in these equations) are discarded. Instead a phenomenological source-term that is concentrated in the surface layers, is used for the poloidal field equation. This term is parameterized to be mathematically similar to the αB term in the dynamo equations, but in spirit is very different as it does not involve averaging over small-scale helical turbulence, neither does it invoke the first-order smoothing approximation. A suitable computational algorithm for buoyancy is also prescribed that is used to mimic the buoyant eruption of toroidal flux tubes (Nandy & Choudhuri 2001). Meridional circulation plays an important role in these class of models in transporting the poloidal field first poleward and then downward into the tachocline where the strong toroidal fields are stored and amplified; the counterflow in the circulation is also responsible for the equatorward propagation of the sunspot formation belt. The meridional flow speed is believed to control the sunspot cycle period in these class of models (Charbonneau & Dikpati 2000; Nandy 2004; Hathaway et al. 2004; Yeates, Nandy & Mackay 2008). In Figure 4, we present a simulated butterfly diagram from one such dynamo model powered by the Babcock-Leighton idea for poloidal field generation. We see that the large-scale features of the solar cycle are fairly well-reproduced in by this class of solar dynamo models.

5 Concluding Remarks

My endeavor here has been to give a flavor of ideas and concepts in dynamo theory. Given that this is directed towards students, who are possibly being exposed for the first time to dynamo theory, I have concentrated on highlighting important concepts in MHD. I have also laid down a pedagogical account of the development of ideas in this subject, stressing particulary on its mathematical formulation. Finally, I have focussed on the solar cycle as a representative case study to illustrate how the theoretical ideas come together to explain the solar dynamo mechanism. It has to be stressed however that this is just a limited account of the subject. Those desirous of a serious pursuit of dynamo theory are encouraged to consult other related works that give a more detailed account of this subject. Amongst these are the books “Magnetic Field Generation in Electrically Conducting Fluids” (Moffatt 1978), “Cosmical Magnetic Fields” (Parker 1979), “The Physics of Fluids and Plasmas: An Introduction for Astrophysicists” (Choudhuri 1998) and the recent reviews on the solar dynamo mechanism by Ossendrijver (2003) and Charbonneau (2005). This effort will be well served if it is only the beginning, and inspires a more careful and sustained effort at studying dynamo theory and perhaps researching some of its outstanding issues; there are many!

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