2D MHD model of the solar corona and solar wind: Recent results

E. C. Sittler Jr.[1] and L. Ofman[2, 1]

[1]NASA Goddard Space Flight Center, Greenbelt, MD, USA
[2]The Catholic University of America, Washington, D. C., USA

Abstract. We have been developing a self-consistent 2D MHD steady-state model of the solar corona and solar wind. This model is empirically constrained by 2D maps of the effective temperature, T_{eff} , and heat flux, q_{eff} , as originally derived by Sittler and Guhathakurta or SG model. The SG model has been applied to Mark II K-coronameter data, Skylab, Spartan and SOHO/LASCO white light coronagraph data, plus plasma and magnetic field Ulysses data. Our model uses the MHD conservation equations of mass, momentum and energy with a multipole expansion of the coronal magnetic field up to octupole term as the initial state for the MHD solutions. At present our MHD solutions are confined between the coronal base at 1.03 R_S and 5 R_S and will be expanded to larger heliocentric distances in the future. In order to make our MHD solutions more tractable, we have fit smooth analytical functions to T_{eff} and q_{eff} derived from the data-driven SG model. We will present solutions under these conditions, some of the difficulties we have had to deal with and show the future direction of our research.

Index Terms. MHD, solar corona, solar wind.

---

1. Introduction

We are developing self-consistent 2D MHD steady-state solutions of the solar corona and solar wind using empirically determined estimates of the effective temperature T_{eff} and effective heat flux q_{eff} . Our modeling has concentrated its efforts on the Sun's corona during solar minimum when the corona and solar wind are relatively simple to model, azimuthal symmetry is a fairly good approximation and the corona is in a quasi-steady state configuration. This modeling effort has been focused around the semi-empirical model originally presented by Sittler and Guhathakurta (1999a) for which we will refer to as SG for short. Eventually, we plan to generalize toward a more ambitious program of modeling the corona in 3D using, for example, magnetogram data as a boundary condition of our solutions. During solar maximum the boundary conditions are more complex and will require observations of sufficient precision that may not be available at this time. STEREO, Solar-B and SDO may allow this to be done with some success in the future. Even present 3D MHD codes, which are supposed to be more exact, are unable to provide realistic solutions of the coronal plasma and magnetic field. It is our scientific opinion that such models are flawed since wave pressure terms and energy flow terms are not known. Because of this, many modelers do not include the energy equation for their solutions or ad hoc momentum and energy deposition terms are used. The use of the empirically determined effective temperatures and effective heat flux will allow closure of the MHD equations.

As we will show, our modeling effort can produce convergent solutions with high speed flows over the poles and a very complex magnetic field structure within the equatorial plane. Our modeling effort uses more realistic magnetic field topologies for the initial state, such as an octupole field rather than dipolar configurations, used in many previous models, which gives a poor description of the Sun's coronal field. But, our modeling effort has also shown that use of a monopole term near the equator may not yield a sufficiently accurate boundary condition for our self-consistent solutions. To better define our boundary conditions at the base of the corona we will eventually use potential magnetic fields determined from magnetogram data from SOHO or Earth based observations.

2. Thermally conductive MHD model

The basic equations for our self-consistent calculations are described in Sittler et al. (2003) and Ofman (2004) and we refer to these equations as the Thermally Conductive MHD Model. They are as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \vec{V} \right) = 0 \tag{1}$$

$$\rho \left[\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \bullet \nabla \right) \vec{V} \right] = -\nabla p - \frac{G M_S \rho}{r^2} + \frac{1}{c} \vec{J} \times \vec{B} + \vec{F}_{\nu} + \vec{P}_i \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} = -c\nabla \times \vec{E} \quad (3)$$

$$\vec{E} = -\frac{1}{c}\vec{V} \times \vec{B} + \eta \vec{J} \quad (4)$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} \quad (5)$$

$$\frac{\partial T}{\partial t} = -(\gamma - 1)T\nabla \bullet \vec{V} - \vec{V} \bullet \nabla T + (\gamma - 1)(H_C / \rho + H_i) \quad (6)$$

$$H_C = \nabla\left[\xi T^{5/2} \frac{\nabla T \bullet \vec{B}}{B^2}\right] \bullet \vec{B} \quad (7a)$$

$$H_i = H_0(r-1)e^{-r/\lambda}h(\theta) \quad (7b)$$

$$P_i = P_\theta f(r,\theta) \quad (7c)$$

As can be seen this model does not assume a polytrope, but instead uses the energy equation given by equations 6 and 7. In the paper by Sittler et al. (2003), they considered three cases, all of which used the multipole expansion of the magnetic field as determined by SG, which included a dominant octupole term. The model did not use a monopole term for our initial boundary conditions at the base of the corona. The first case, case A, they did assume a polytrope with γ = 1.05. In case B, they included the heat conduction term and ad hoc heating term. Finally, in case C, they used the semi-empirically determined effective heat flux from SG which was incorporated into the heating term

$$H_{i,eff} = -\vec{B} \bullet \nabla \frac{q_{eff}}{B} \quad (8)$$

and by setting the heat conduction term H_C = 0. In all three cases the self-consistently computed magnetic field topology displayed three helmet streamers with outflow velocities V ~ 300 km/s at 5 R_S for cases A and B. In case C the flow speed rose rapidly inside 2 R_S to about V ~ 150 km/s, but then

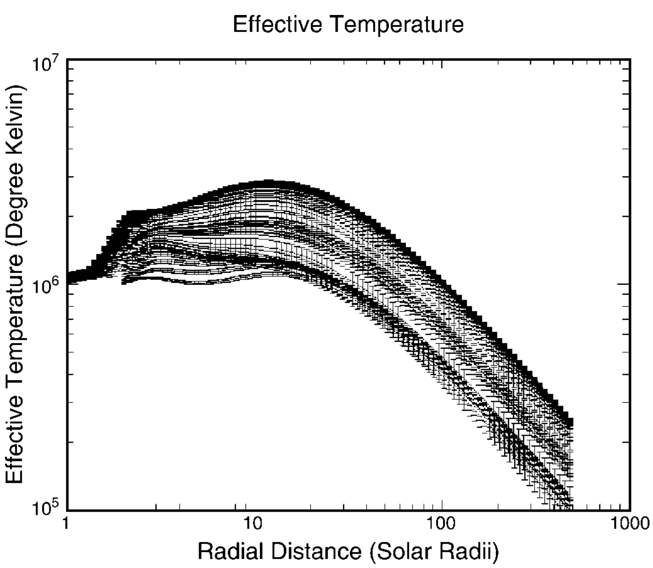

Fig. 1. Plot of T_{eff} for 2D error analysis study. Error bars are shown.

drooped down to about V ~ 115 km/s at 5 R_S. As we later show, this effect indicated the need for extended momentum addition and the inclusion of T_{eff} into the momentum equation. The results presented in Sittler et al. (2003), definitely supported the hypothesis by Sittler and Guhathakurta (1999b) that the octupole field requires a three current sheet topology in order to remove the disconnected field lines present in the solutions by SG. A central flaw in these solutions is the fact that the latitude of the streamers are ~ 45°, which is contrary to observations by Guhathakurta and Holzer (1994) and later used by SG where the polar coronal hole should dip down to a latitude ~ 27° outside r ~ 2 R_S.

3. Error analysis and empirical model of T_{eff} and q_{eff}

Our approach to reconcile the problem of helmet streamer location and the drooping of the outflow velocities over the poles was to introduce T_{eff} into the momentum equation and incorporate a monopole term as a boundary condition for the magnetic field at the base of the corona. Our initial attempts led to unstable solutions near the equatorial plane, where the field became very complex and eddies were found to form and be ejected away from the Sun. The eddies were similar to the small blobs of plasma observed in the LASCO data and used by Shelley et al. (1997) to measure slow solar wind speeds near the equatorial plane where the blobs were observed. It is also in the equatorial plane where coronal mass ejections are observed (Linker et al., 2003) in a semi-random fashion and furthermore the slow solar wind is observed to be more time dependent. It could be that the actual equatorial corona is marginally stable. To help reconcile this problem we realized that by incorporating the semi-empirical determinations of T_{eff} and q_{eff} into our self-consistent calculations we needed to perform numerical derivatives of the “data”. This may have introduced some numerical uncertainty to the solutions. Therefore, we decided to perform an error analysis of the SG results for the density, magnetic field model, T_{eff} and q_{eff}. We then fit a model to the 2D maps of T_{eff} and q_{eff}, which could then be put into our self-consistent calculations, to remove any numerical instabilities and achieve convergent solutions. The result of this error analysis is shown below for the density model and magnetic field model with parameters given in Table 1 with σ errors:

Empirical density model:

$$N_e(r,\theta) = N_p(r) + [N_{cs}(r) - N_p(r)]\exp(-\lambda^2 / w^2(r))$$
$$N_k(z) = a_{k1}\exp(a_{k2}z)z^2 P_k(z)$$
$$P_k(z) = 1 + a_{k3}z + a_{k4}z^2 + a_{k5}z^3; k = p, cs$$

Empirical magnetic field model:

$$\Psi = \frac{z^3}{4}((6-5\cos^2\theta)\cos^2\theta - 1) + \eta_Q z^2 \sin^2\theta\cos\theta$$
$$+ \eta_D z \sin^2\theta + \eta_M (1-\cos\theta)$$

We then performed a whole series of solutions for which we varied the ±σ errors for the density and magnetic field parameters independently of each other and compiled ~ 30 solutions from which we could compute mean and ±σ errors for the effective temperature and heat flux in 2D space. These results are shown in Figs. 1 and 2. The results are quite revealing showing both the radial and latitudinal dependences of the solutions. The higher T_{eff} and q_{eff} curves occur at

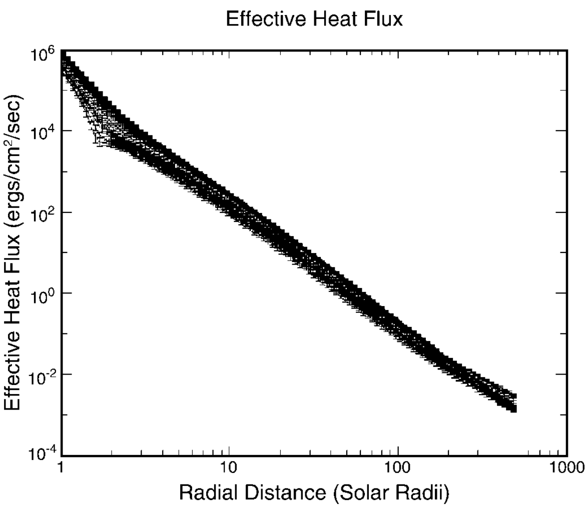

Fig. 2. Plot of q_{eff} as a function of radial distance for the full range of latitudes covered by the SG model. Error bars are shown.

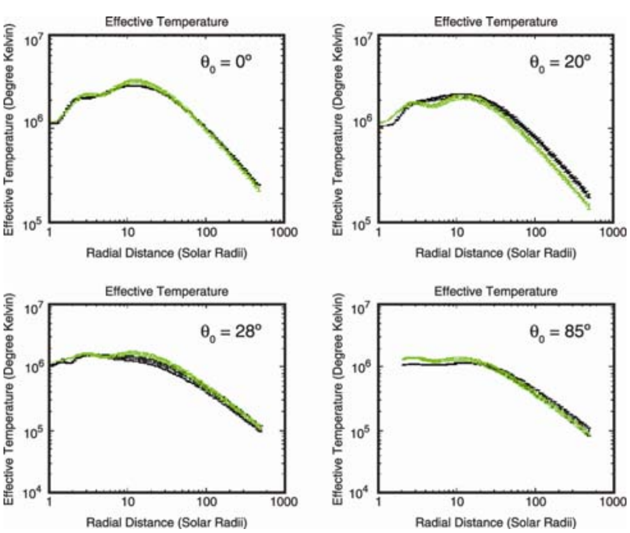

Fig. 3. Shows quality of model fits to T_{eff} 2D maps. The four panels are for θ = 10°, 20°, 28° and 85°. Black lines are data, green lines are fit.

higher latitudes where high speed flows occur. We then needed to construct an empirical mathematical model to describe the complex structure displayed in Figs. 1 and 2. To do this we used as our starting point the solutions for a spherically symmetric model with density $N_e = N_0 z^2 e^{\alpha z}$ where z = 1/r. This solution can be computed in closed form (Sittler, 1978). We then had to add a correction term to model the initial rise in T_{eff} over the poles for r < 2 R_S. In Figs. 3 and 4 we show the quality of the model fits to the T_{eff} and q_{eff} 2D maps. The model functions organize the data quite well. The fits will provide the parameter range, within ±σ levels of T_{eff} and q_{eff} parameter determinations so that we can adjust the parameters to achieve convergent solutions.

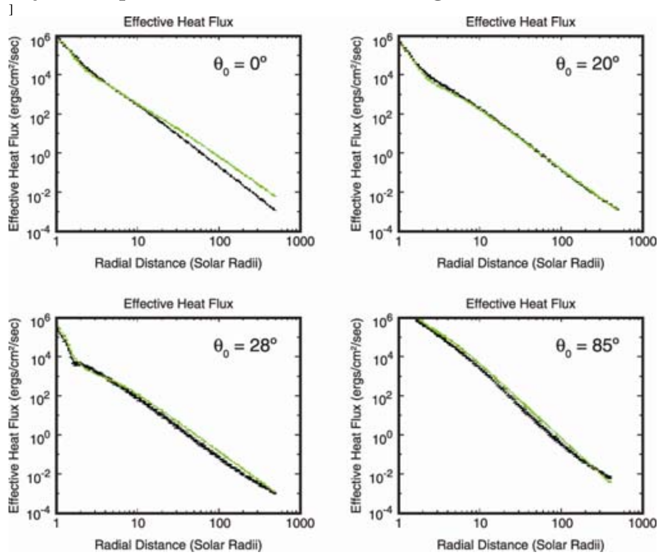

Fig. 4. Shows quality of model fits to q_{eff} 2D maps. The four panels are for θ = 10°, 20°, 28° and 85°. Black lines are data, green lines are fit.

4. Preliminary convergent solutions and future work

We have been able to achieve such solutions with some degree of success. An example, of such a solution is shown in Fig. 5. This solution, which displays just the northern quadrant, shows that we have a three streamer solution, with high speed flows over the poles as expected with V ~ 400 km/s at r ~ 5 R_S. But, the solutions also show a pealing off of magnetic flux at the tops of the streamers indicating a very complex structure near the equatorial plane. As previously discussed, such time dependent features have been found to occur in SOHO LASCO data (Sheeley et al., 1997). In order to achieve these solutions we had to weaken the monopole

Table 1. Empirical Density and Magnetic Field Parameters

Parameter	Value	One Sigma Error
a_{cs1}	3.292×10^{-3}	7.12×10^{-5}
a_{cs2}	6.496	0.145
a_{cs3}	5.12	0.32
a_{cs4}	-8.25	0.49
a_{cs}	3.95	0.365
a_{p1}	1.2824×10^{-3}	3.144×10^{-5}
a_{p2}	4.2761	0.0631
a_{p3}	4.682	0.197
a_{p4}	-22.06	0.486
a_{p5}	29.981	0.878
η_M	0.282	0.045

η_D	0.109	0.142
η_Q	0.107	1.034

term relative to that determined by SG. But, the addition of the monopole term has caused the off equatorial plane streamers to tilt down near the equatorial plane as observed. Therefore, it does indicate that we can get solutions more characteristic of observations. For future work we would like to continue this analysis, but use magnetic fields determined from magnetogram data to better define our boundary conditions at the base of the corona and achieve more realistic solutions. We also need to cover a broader range of parameter space for our analytical fits to the 2D maps of T_{eff} and q_{eff}. These results, once completed, will establish a self-consistent 2D MHD model that correctly describes all the observational parameters of the plasma and field. With a true representation of the coronal magnetic field and plasma at hand, one would then be in a better position to start investigating the various transport coefficients due to waves or electron heat flux. For example, we could compare the three-fluid theoretical calculations by Ofman and Davila (2001), with our single fluid empirical estimates of wave pressure and energy flow terms to construct equivalent transport terms for our MHD simulations. A further enhancement of the physics could be achieved by introducing a field aligned electric field into our simulations by using our semi-empirical model of electron heat flux described in Sittler et al. (2005). In the long term, we would want to perform realistic time dependent solutions (i.e., our solutions are essentially time dependent, but solve those cases where convergence occurs to a steady state solution) for the corona and solar wind and study its susceptibility to configuration space and velocity space instabilities. We would then be in a good position to develop realistic models of the instabilities that give rise to CMEs, and to model the propagation of CMEs through the corona, that will include realistic plasma heating.

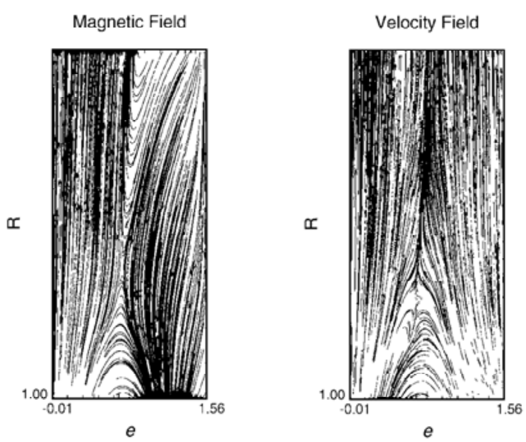

Fig. 5. The magnetic field vectors, left panel, and velocity vectors right panel, in the upper right corner of the corona**.** The solutions were obtained with T_{eff} from Sittler and Guhathakurta (1999) being used in the momentum equation. Contrary to thermally conductive MHD model, the field lines at the coronal boundary now dip to lower latitudes as observed.

5. Conclusions

The model of SG provides an empirical 2D MHD description of the corona and the solar wind. However, the solutions are not self-consistent. The model favors octupole field geometry.

Furthermore, SG results give a poor description inside r ~ 2 R_S at low latitudes, where field lines become disconnected. Later models with multiple current sheets help alleviate the problem. Using the thermally conductive 2D MHD model with our empirically determined magnetic field and q_{eff}, we are able to get solutions with multiple current sheets, but current sheets were at $\lambda \sim 45°$ and did not merge with equatorial current sheet for r > 2-3 R_S as observed. See Sittler et al. (2003). Thermally conductive 2D MHD model with q_{eff} alone does not yield high speed wind, contrary to observations over the poles. When 'raw' T_{eff} is used instead of T we encounter numerical stability problems, so we developed analytical models of T_{eff} and q_{eff} that were then fit to "empirical data" from SG. The use of analytical models improved numerical stability of solutions (see Fig. 5). We plan to improve fitting algorithm for analytical models, by improving the initial guess of the functions. Enhancements to the initial magnetic field model may be required, especially when it comes to monopole term which tends to cause disconnected field lines (i.e., use potential fields with magnetogram data). We will investigate the stability of various solutions, and numerical methods in the future.

Acknowledgements: The authors would like to thank support by NASA Grant NAG5-11815.

References

M. Guhathakurta and T. E. Holzer, "Density structure inside a polar coronal hole", *Astrophys. J.*, vol. 426, pp. 782-786, 1994.

J. A. Linker, Z. Mikic, R. Lionello and P. Riley, "Flux cancellation and coronal mass ejections", *Phys. Plasmas*, vol. 10, pp. 1971-1978, 2003.

L. Ofman and J. M. Davila, "Three-fluid 2.5 dimensional magnetohydrodynamical model of the effective temperature in coronal holes", *Astrophys. J.*, vol. 553, pp. 935-940, 2001.

L. Ofman, S. P. Gary and A. F. Vinas, "Resonant heating and acceleration of ions in coronal holes driven by cyclotron resonant spectra", *J. Geophys. Res.*, vol. 107, p. 1461, 2002.

L. Ofman, "The origins of the slow wind in coronal streamers", *Adv. Space Res.*, vol. 33**,** pp. 681-688, 2004.

N. R. Sheeley et al., "Measurement of flow speeds in the corona between 2 and 30 R_{SUN}", *Astrophys. J.*, vol. 484, pp. 472-478, 1997.

E. C. Sittler Jr., "Study of the electron component of the solar wind and magnetospheric plasma", *Ph.D. Thesis*, Massachusetts Institute of Technology, 1978.

E. C. Sittler, Jr. and M. Guhathakurta, "Semiempirical two-dimensional magnetohydrodynamic model of the solar corona and interplanetary medium", *Astrophys. J.*, vol. 523, pp. 812-826, 1999a.

E. C. Sittler, Jr. and M. Guhathakurta, "Proc. of Solar Wind 9 Conference", in *AIP Conference Proceedings*, vol. 471, S. Habbal, Ed. Nantucket MA, 5-9 October 1998, 1999b, pp. 401-404.

E. C. Sittler Jr., L. Ofman, S. Gibson, M. Guhathakurta, J. Davila, R. Skoug, A. Fludra and T. Holzer, "Development of Multidimensional MHD model for the solar corona and solar wind", Proc. Solar Wind X, Pisa Italy, June 2002, in *AIP Conference Proceedings*, vol. 679, AIP, NY, 2003, pp. 113-116.

E. C. Sittler Jr., R. Skoug, L. Ofman and A. Fludra, "Semi-empirical model of electron heat heat flux: SOHO and Ulysses observations", in *Proc. Solar Wind 11 – SOHO 16 "Connecting Sun and Heliosphere", Whistler, Canada, 12-17 June 2005 (ESA SP-592, September 2005)*, B. Fleck and T. H. Zurbuchen, Ed. 2005, pp. 445-448.